Technical Comments

Comment on "A Method of Earth-Pointing Attitude Control for Elliptic Orbits"

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THE pitch stability charts of an Earth-pointing, moment-of-inertia controlled satellite presented by Connell[1] for elliptic orbits are not correct. The pitch equation

$$\theta'' + g\theta' + 3\{[(k-1)(1+e\cos\psi)^2 + (1+e)^2]/[(1+e)^2(1+e\cos\psi)]\}\theta = 0 \quad (1)$$

given as Eq. (12) in Ref. 1 can be reduced for small eccentricities $e \ll 1$ and $g = 0$ to the Mathieu equation

$$\theta''(\lambda + \gamma\cos\psi)\theta = 0 \quad (2)$$

where $\lambda = 3k(1-2e)+6e$ and $\gamma = 3(k-2)e$. Using the well-known results for the stability boundaries of the Mathieu equation for small periodic terms $\gamma \ll 1$; e.g., given by Magnus,[2]

$$\lambda_1 = 0, \quad \lambda_{2,3} = \tfrac{1}{4} \pm \tfrac{1}{2}\gamma, \quad \lambda_{4,5} = 1, \quad \lambda_{6,7} = \tfrac{9}{4} \quad (3)$$

it follows immediately that the stability boundaries for satellites in slightly elliptic orbits are

$$k_1 = -2e, \quad k_2 = \tfrac{1}{12} - \tfrac{7}{8}e, \quad k_3 = \tfrac{1}{12} - \tfrac{67}{24}e$$
$$k_{4,5} = \tfrac{1}{3} - \tfrac{4}{3}e, \quad k_{6,7} = \tfrac{3}{4} - \tfrac{1}{2}e \quad (4)$$

Therefore, the pitch stability chart, Fig. 2 of Ref. 1, obtained for $g = 0.3$ and $g = 1.0$ do not correspond to subregions of Fig. 1 of even in the case of arbitrary eccentricities, this pitch stability chart is only qualitatively correct. For comparison, the pitch stability chart for $g = 0$, found in a prior investigation,[3] is shown in Fig. 1 of this comment.

For the general case $g > 0$ two properties can be stated following from the theory of ordinary differential equations; e.g., Cesari[4]: 1) The instability regions for $g > 0$ are subregions of instability regions for $g = 0$, 2) The time constant of the pitch control system is

$$T = \left(\frac{1}{2}\int_0^{2\pi} g\, d\psi\right)^{-1} = (g\pi)^{-1} \quad (5)$$

The pitch stability charts, Figs. 3 and 4 of Ref. 1, valid for $g = 0.3$ and $g = 1.0$ do not correspond to subregions of Fig. 1 of this Comment. A pitch stability chart for the pitch angular motion with a linear feedback control law similar to that used in Ref. 1 can be found in Ref. 5 to confirm property 1.

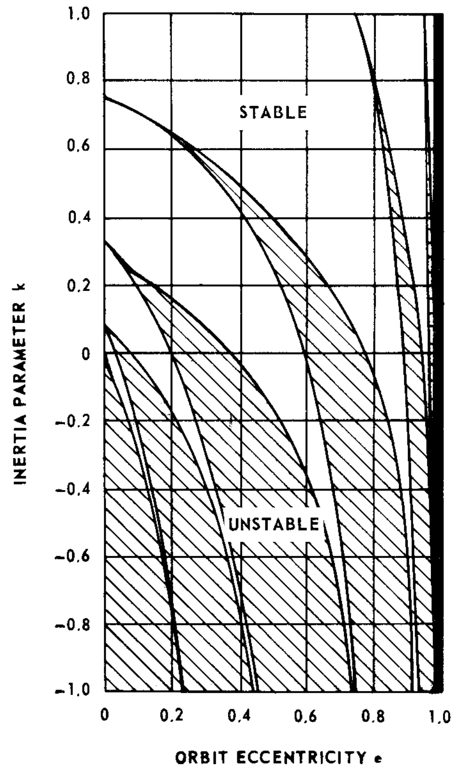

Fig. 1 Pitch stability chart, $g = 0$.

The time constants shown in Fig. 5 of Ref. 1 are correct only for small eccentricities. Actually, they should be completely independent of eccentricity e and inertia parameter k because of property 2. Especially incorrect is the statement in Ref. 1 that in the case $g = 0$ asymptotic stability will occur. The stability region in Fig. 1 of this Comment represents stability in the sense of Liapunov. This is obvious not only by property 2 above but also by the fact that the Hill equation determined from Eq. (1) for $g = 0$, possesses only stable and unstable solutions. The fictitious asymptotic stability found in Ref. 1 may be caused by rounding errors in the numerical computation which, based on past work, increase with eccentricity.

References

[1] Connell, G. M., "A Method of Earth-Pointing Attitude Control for Elliptic Orbits," *AIAA Journal*, Vol. 10, No. 3, March 1972, pp. 258–263.

[2] Magnus, K., *Schwingungen*, 2nd ed., Teubner, Stuttgart, 1969.

[3] Schiehlen, W., "Über die Lagestabilisierung künstlicher Satelliten auf elliptischen Bahnen," Ph.D. thesis, March 1966, Univ. of Stuttgart, Germany.

[4] Cesari, L., *Asymptotic Behavior and Stability Problems in Ordinary Differential Equations*, 2nd ed., Academic Press, New York, 1963.

[5] Schiehlen, W., "Dämpfung erzwungener Schwingungen von Satelliten auf elliptischen Bahnen," *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 51, 1971, pp. T210–T211.

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